Cylindrical Dielectric Resonators and Their Applications in TEM Line Microwave Circuits

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Abstract—**A cylindrical sample of low-loss high ϵ_r dielectric placed between two parallel conducting plates perpendicular to the sample axis forms a microwave resonator. A simple approximate method for predicting the resonant frequencies of the TE modes of this structure is developed. The method becomes exact for the limiting case of this structure which is known as a dielectric post resonator. In all cases, the accuracy of the method is shown to be better than 3.5 percent. The $TE_{01\delta}$ mode chart presented allows the determination of the resonant frequency and the tuning range of any cylindrical dielectric resonator for which $\epsilon_r \geq 10$. The properties of the dielectric resonator as a TEM line element are demonstrated experimentally.**

I. Introduction

DIELECTRIC resonators exhibiting high Q factors and very low temperature dependence of the resonant frequency have been recently reported [1], [2]. They offer waveguide cavity performance with great reduction of cost and size. Also, they have proven to be useful elements of microwave band-reject and bandpass filters [1], [3]–[8], slow-wave structures [9], up- and downconverters [10], diode and FET oscillators [11], [12], and frequency selective limiters [13]. They are also compatible with TEM line circuits. Therefore, the use of dielectric resonators in microwave circuits can be expected to expand rapidly.

The dielectric resonators of cylindrical shape excited in the $TE_{01\delta}$ mode were used in most of the previously mentioned applications. In the past, many attempts were made to calculate the resonant frequency of this mode [5], [14]–[22]. However, none of them was sufficiently accurate and/or simple enough to serve for design purposes. The resonant frequencies computed using a magnetic wall waveguide model [5], [14]–[17] are about 4 to 10 percent smaller than the measured values. On the other hand, the dielectric waveguide model [18], [19] results in an error up to 7 percent as demonstrated in this paper. The method recently reported by Konishi *et al.* [20] gives results in very close agreement with the dielectric waveguide model as shown by Itoh and Rudokas [19]. The most recent method published by Guillon and Garault is a mixture of the magnetic wall and the dielectric waveguide models [21]. This method, however, is very complicated and gives little insight into the actual field distribution of the $TE_{01\delta}$ mode in the dielectric resonator. Its 1-percent accuracy was demonstrated for resonators in which the effect of metallic walls is negligible. This error can be expected to be larger for resonators in which the effect of metallic walls is significant.

Manuscript received July 12, 1977; revised March 13, 1978.

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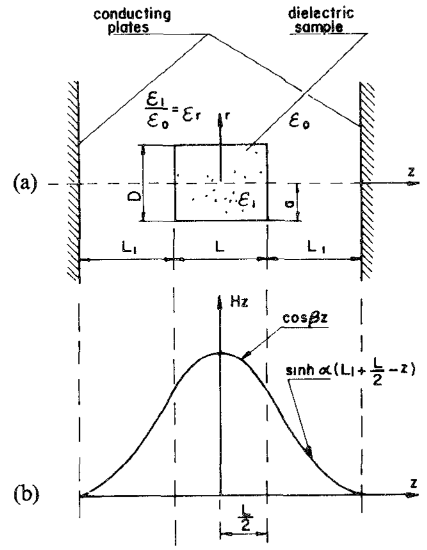

Fig. 1. (a) The dielectric resonator structure. (b) H_z field distribution along z axis for the $TE_{01\delta}$ mode.

This paper deals with TE modes of dielectric resonators composed of an isotropic cylindrical sample between two parallel perfectly conducting plates perpendicular to the sample axis as shown in Fig. 1. The analytic approach taken is a modification of the dielectric waveguide model [18], [19], which takes into account the change of cross-sectional field distribution along z axis. With this approach one can give qualitative explanation of the reported contradictions between various measured field distribution in cylindrical dielectric resonators [18], [23]. The other advantages of the method presented here are as follows.

The resonant frequencies are predicted with an error of less than 3.5 percent for any distance between the metallic plates and the dielectric sample. If the distance between the metallic plates and the sample is equal to zero, the approximate solution becomes the exact solution for the

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dielectric post resonator [24]. The introduction of the normalized resonant frequency

$$F_0 = \frac{\pi D}{\lambda_0}\sqrt{\epsilon_r}$$

allows the design equations to be put in a form that has a solution for $\epsilon_r \rightarrow \infty$. This results in the $TE_{01\delta}$ mode chart presented in this paper from which the resonant frequency and tuning range of any cylindrical dielectric resonator with $\epsilon_r \geqslant 10$ can be determined. Therefore, the theory is now useful for design purposes. The theory is supported by experimental data reported by Cohn [5] and other data obtained by the author.

Some illustrative experiments were performed which demonstrate the compatibility of dielectric resonators with TEM line circuits. These include examples of reaction-type cavity and transmission-type cavity. The examples of dielectric resonator—TEM line narrow-band, band-reject, and bandpass filters—were previously described [25].

II. Resonant Frequency for TE Modes

$T_{01\delta}$ Mode—Symmetrical Structure

The resonant structure under consideration is shown schematically in Fig. 1. The following analysis is based on three assumptions.

1) The dielectric sample can be treated as a lossless cylindrical dielectric waveguide of length L excited in the TE_{01} mode [26].

2) The z-dependence of the magnetic field z-component for $|z| \geqslant \frac{1}{2}L$ can be described by $\sinh \alpha(L_1 + \frac{1}{2}L \pm z)$.

3) The cross-sectional field distribution of H_z for $|z| \geqslant \frac{1}{2}L$ is the same as for the TE_{01} mode in cylindrical dielectric waveguide at the "cutoff" frequency (as opposed to metallic waveguides, there is no discrete mode solution in dielectric waveguides below the "cutoff" [27]).

For convenience we define the normalized resonant frequency in the following way:

$$F_0 = \frac{\pi D}{\lambda_0}\sqrt{\epsilon_r} \tag{1}$$

where λ_0 is the free-space wavelength corresponding to the resonant frequency f_0, D is the diameter, and ϵ_r is the relative dielectric constant of the sample.

From the first assumption, F_0 must satisfy the following equations [26]:

$$\frac{J_1(u)}{uJ_0(u)} = -\frac{K_1(w)}{wK_0(w)} \tag{2}$$

$$F_0^2 = (u^2 + w^2)\frac{\epsilon_r - 1}{\epsilon_r} \tag{3}$$

where J_n is Bessel function of the first kind of nth order and K_n is modified Hankel function of nth order. The arguments w and u are defined by equations

$$\left(\frac{u}{a}\right)^2 = \left(\frac{F_0}{a}\right)^2 - \beta^2 \qquad \left(\frac{w}{a}\right)^2 = \beta^2 - \left(\frac{F_0}{a}\right)^2 \frac{1}{\epsilon_r} \tag{4}$$

TABLE I
The Measured and Computed Resonant Frequencies ($TE_{01\delta}$ Mode; Structure as in Fig. 1; $\epsilon_r = 36.2$)

D (mm)	L (mm)	L_1/L	fo (GHz) computed ref (17)	ref. (19)	eqs (2),(3),(7)	measur.
4.06	5.15	0.568	10.09	10 86	10.82	10.48
6.03	4.16	0.820	7.42	8.31	8.20	7.94
5.98	2.96	1 36	8.03	9 16	8.94	8.64
6.02	2.14	2.07	8.70	10.08	9.71	9.40
7 99	2.14	2.07	7.16	8.38	7.96	7.79

where β is the propagation constant of the mode in cylindrical dielectric waveguide and a is the radius of dielectric cylinder.

From the second assumption, α and β must satisfy the following transcendental equation:

$$\cot \beta \frac{L}{2} = \frac{\beta}{\alpha} \tanh \alpha L_1. \tag{5}$$

From the third assumption, α is given by the following expression:

$$\alpha^2 = \left(\frac{\rho_{01}}{a}\right)^2 - \frac{1}{\epsilon_r}\left(\frac{F_0}{a}\right)^2 \tag{6}$$

where ρ_{01} is the first root of J_0. Equations (4), (5), and (6) can be combined into one to obtain

$$\tan\left(\frac{L}{D}\sqrt{F_0^2 - u^2}\right) = \frac{\sqrt{\rho_{01}^2 - \frac{1}{\epsilon_r}F_0^2}}{\sqrt{F_0^2 - u^2}\,\tanh\left(2\frac{L_1}{D}\sqrt{\rho_{01}^2 - \frac{1}{\epsilon_r}F_0^2}\right)}. \tag{7}$$

The normalized resonant frequency F_0 can be computed from the set of (2), (3), and (7) as a function of D/L, L_1/L, and ϵ_r.

The first two assumptions of the preceding analysis are identical with the assumptions underlying the analysis by Itoh and Rudokas [19]. However, to obtain the expression for α they match the tangential field components at the ends of the dielectric cylinder ($z = \pm \frac{1}{2}L$, $r \leqslant a$), as the matching over the whole planes $z = \pm \frac{1}{2}L$ is not possible. In the preceding analysis, the sudden change of the cross-sectional field distribution at the planes $z = \pm \frac{1}{2}L$ has been allowed. It approximates to some extent the change of the cross-sectional field distribution that exists in a real resonator. There has been no experiment performed to confirm this hypothesis directly. However, it removes the contradictions between the field distribution measurements reported by Chow [18] and D'Aiello [23] and also, as we shall show, provides much better agreement between measured and computed values of resonant frequencies.

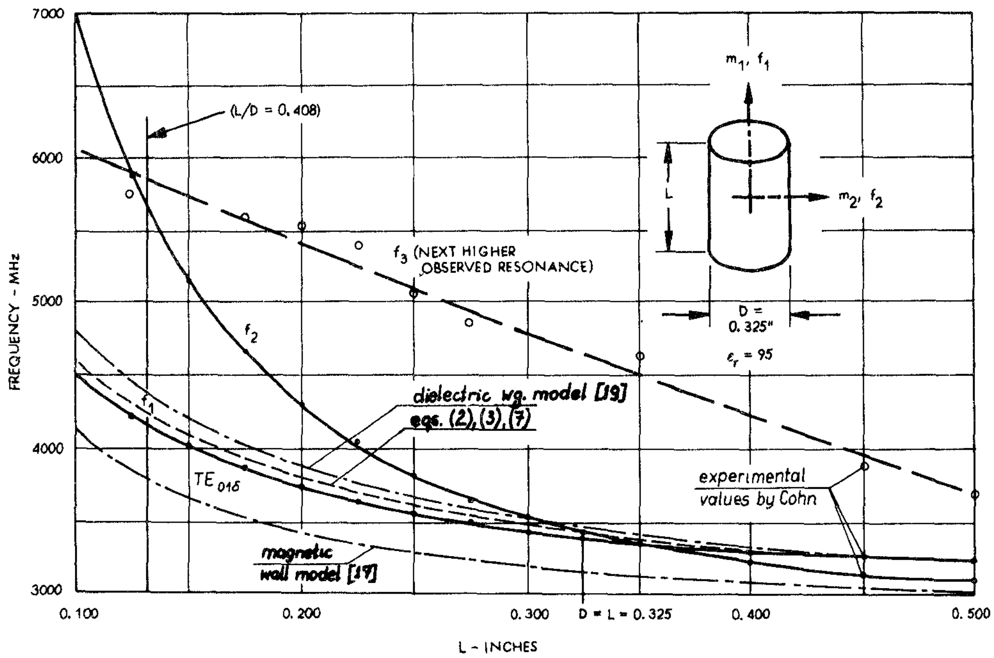

Fig. 2. The comparison of Cohn's experiment [5] with computations based on different models.

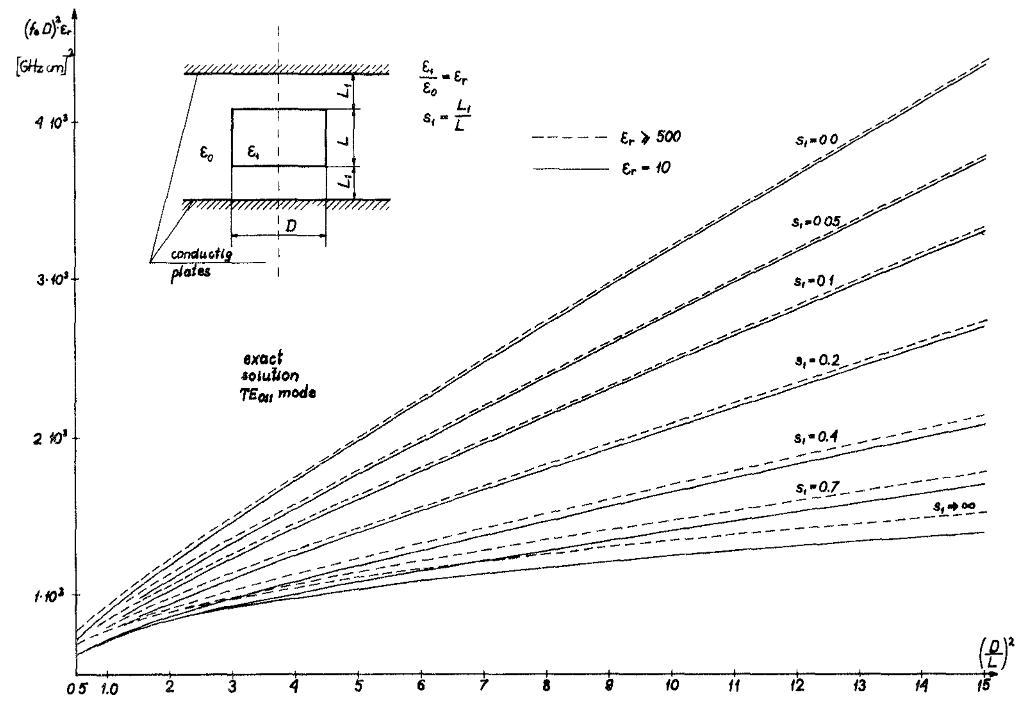

Fig. 3. The $TE_{01\delta}$ mode chart.

The comparison between the resonant frequencies of the $TE_{01\delta}$ mode computed by different methods and the experimental results are given in Table I and in Fig. 2. The experimental results of Table I are for the structure shown in Fig. 6, and the experimental results in Fig. 2 are those published by Cohn [5]. The relative discrepancy between frequencies measured and computed from (2), (3), (7) is always less than 3.5 percent. For comparison, the error obtained with the magnetic wall waveguide model [17] is 4 to 10 percent, and with the dielectric waveguide model [19] it is up to 7 percent. It was shown experimentally for the structure in Fig. 3 that the polystyrene support affected the resonant frequency by less than 0.1 percent. The permittivity of dielectric samples was measured with an accuracy better than 1 percent by the dielectric resonator method [24], [30], and the sample dimensions were measured with an accuracy better than 0.01 mm.

The dependence of the normalized resonant frequency F_0 on D/L, L_1/L, and ϵ_r is very well illustrated by the mode chart shown in Fig. 3. The dependence of F_0 on ϵ_r is very small for the range $\epsilon_r \geqslant 10$. The solution of (2), (3), (7) in the limit $\epsilon_r \to \infty$ cannot be distinguished graphically from the solution for $\epsilon_r = 500$. The solution for $L_1/L = 0$ is the exact solution for the TE_{011} mode in the dielectric post resonator [24]. Note that the $TE_{01\delta}$ mode chart of Fig. 3 allows the determination of the approximate resonant frequency and the maximum tuning range of any cylindrical dielectric resonator with $\epsilon_r \geqslant 10$. Therefore, this chart is very useful for design purposes.

$TE_{01\delta}$ Mode—Unsymmetrical Structure

An important example of an unsymmetrical dielectric resonator structure is shown in Fig. 4. This is a typical structure used in microwave integrated circuits. The application of the previously described method of analysis results in a set of equations which is formed by (2), (3), and

$$2\frac{L}{D}\sqrt{F_0^2-u^2} = \tan^{-1}\frac{\sqrt{\rho_{01}^2-\frac{1}{\epsilon_r}F_0^2}}{\sqrt{F_0^2-u^2}\,\tanh\left(2\frac{L_1}{D}\sqrt{\rho_{01}^2-\frac{1}{\epsilon_r}F_0^2}\right)} + \tan^{-1}\frac{\sqrt{\rho_{01}^2-\frac{\epsilon_p}{\epsilon_r}F_0^2}}{\sqrt{F_0^2-u^2}\,\tanh\left(2\frac{L_2}{D}\sqrt{\rho_{01}^2-\frac{\epsilon_p}{\epsilon_r}F_0^2}\right)} \tag{8}$$

where ϵ_p is the relative dielectric constant of the substrate. Therefore, F_0 is a function of $D/L, L_1/L, L_2/L, \epsilon_r, \epsilon_p$. As can be seen from Table II, the relative difference between measured and computed values of the resonant frequencies is less than 1.5 percent. This case was also investigated experimentally by Day [28]. The experimental curves he presents are only for one value of D/L_2 and for $D/L_1 = 0$.

Equations (2), (3), and (8) have been used to analyze the tuning properties of resonators placed in the structure as shown in Fig. 5, together with an example of the theoretical and experimental results.

III. Dielectric Resonator as a TEM Line Element

In modern microwave equipment, the TEM lines composed of an inner conductor between parallel conducting plates are very frequently used. But, for many applications, TEM line resonators are less attractive than waveguide cavities. However, dielectric resonators can in-

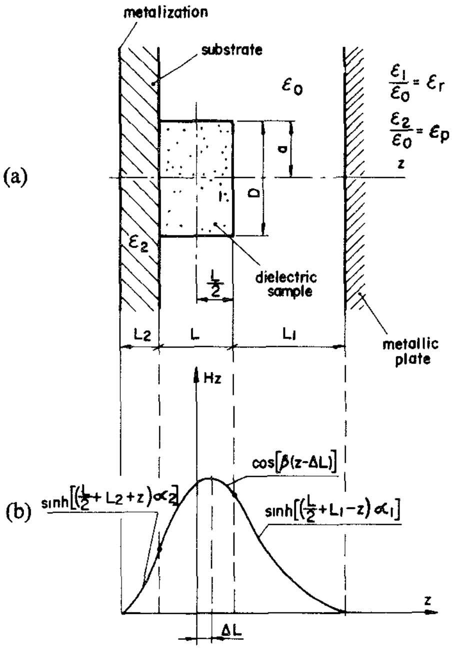

Fig. 4. (a) The dielectric resonator structure typical for MIC. (b) H_z field distribution along z axis for the $TE_{01\delta}$ mode.

TABLE II
THE MEASURED AND COMPUTED RESONANT FREQUENCIES ($TE_{01\delta}$ MODE; STRUCTURE AS IN FIG. 4; $\epsilon_r = 36.2$; $\epsilon_p = 9.5$)

D (mm)	L (mm)	L_1/L	L_2/L	fo (GHz) comput	fo (GHz) measur
6 06	4.22	0.943	0.166	8.37	8 27
6.03	3.04	1.69	0.230	9.18	9.09
6 02	2.14	2.83	0.327	10.26	10.20
7.94	2.10	2.90	0.333	8.83	8.81

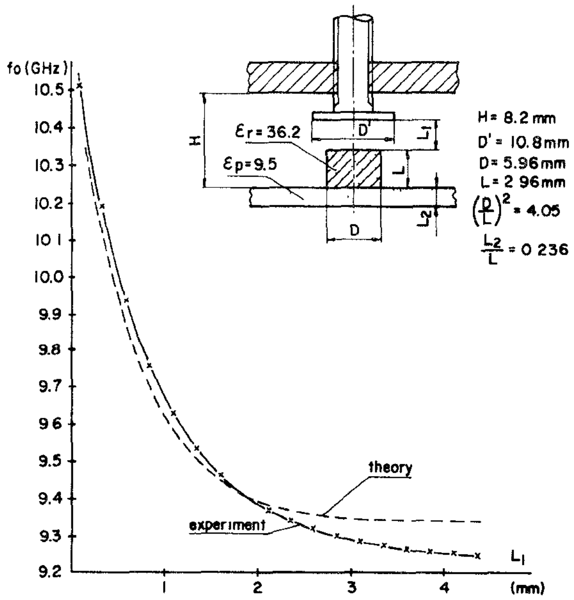

Fig. 5. The tuning of cylindrical dielectric resonators placed in MIC: The cross-sectional view of the structure and the example of theoretical (2), (3), (8), and experimental results.

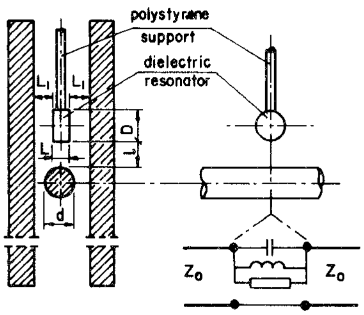

Fig. 6. The dielectric resonator placed in TEM transmission line and its equivalent circuit.

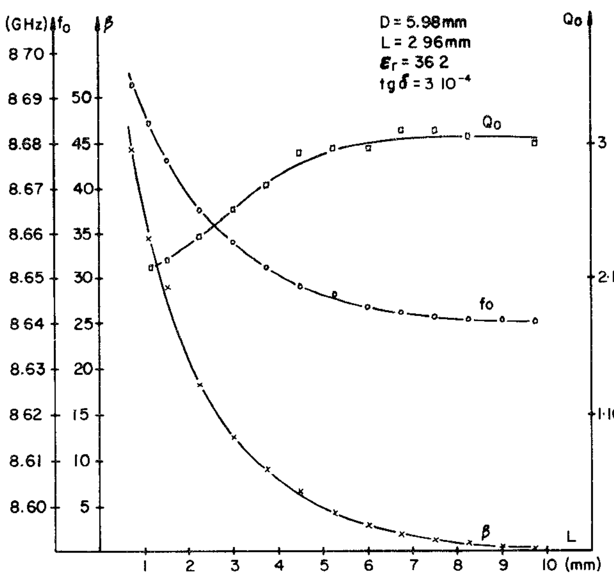

Fig. 7. The dielectric resonator as reaction-type cavity in TEM transmission line of the cross section as in Fig. 6 ($d=6$ mm, $z_0=50\ \Omega$).

troduce the electrical performance and advantages of waveguide cavities into TEM line circuits.

Equivalent Circuit

The cross-sectional view of the structure under investigation is shown in Fig. 6. The cylindrical dielectric sample is placed in a TEM line composed of a round rod between parallel plates. The dielectric sample and parallel plates form a microwave resonator which can be coupled to the TEM wave propagating in the line. In the case of the $TE_{01\delta}$ mode, the coupling is only of the magnetic type as the electric fields of the TEM wave and dielectric resonator are orthogonal. It should be noted that the cylindrical sample is placed in the region of very small electric field intensity; thus it is almost "invisible" to the waves at frequencies much different from the resonant frequency of the resonator. This means that the equivalent circuit of this structure (Fig. 6) for the $TE_{01\delta}$ mode is identical with the circuit of a series-connected reaction-type cavity [29]. Measured data for a particular cavity are shown in Fig. 7. The slight shift in resonant frequency and the deterioration of Q_0 for large coupling coefficients are due to the

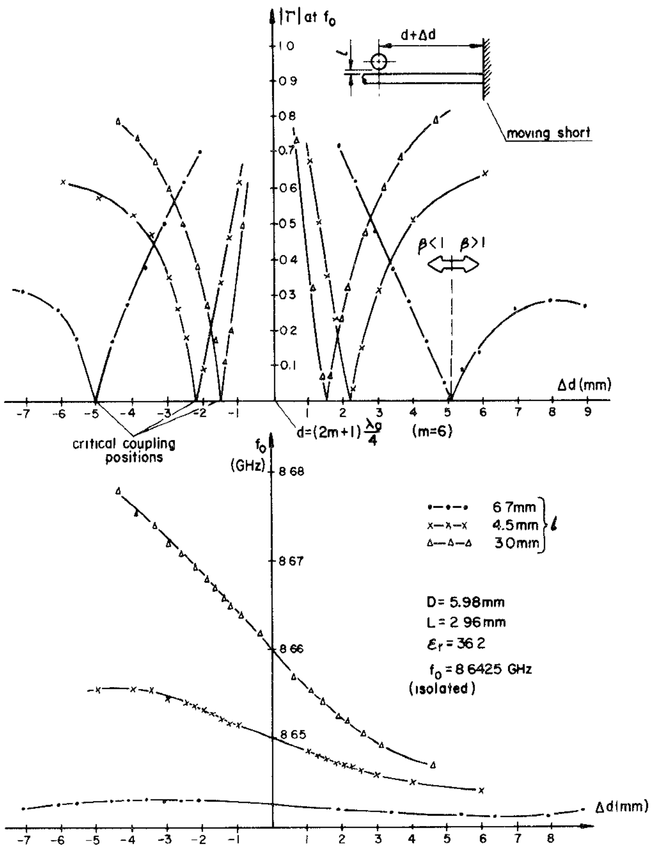

Fig. 8. The dielectric resonator as reflection-type cavity in TEM transmission line of the cross section as shown in Fig. 6 ($d=6$ mm, $z_0=50\ \Omega$).

resonator field perturbation by the inner conductor of the TEM line. For coupling coefficients $\beta \leqslant 30$, the VSWR introduced far from resonance by the presence of the dielectric sample was less than 1.1. The typical frequency stability for the resonator made of TiO_2–ZrO_2 ceramic (manufacturer: UNITRA CERAD, Warsaw, Poland) was $-3\times10^{-6}/°C$ (-0.03 MHz/°C at X-band).

A reflection-type cavity can be constructed by terminating the TEM line in a short circuit, whose position with respect to the sample also determines the cavity coupling. An example of experimental data, taken on the same sample as in Fig. 7, is shown in Fig. 8.

It is interesting to notice that terminating the TEM line with a varactor diode leads to the construction of a reflection-type cavity having electronically tunable coupling. An electronically switched on and off cavity can be constructed with a step-recovery diode.

The experimental data presented here were used for the development of narrow-band, band-reject, and bandpass filters. The realization of these filters and their measured performance have been described elsewhere [25].

IV. Conclusions

Since cylindrical dielectric resonators can be designed to operate with performance similar to a waveguide cavity but with a much smaller size, they offer a very attractive alternative for applications in miniaturized microwave circuits. These resonators can be easily designed by using the curves and equations given in this paper. The resonant frequency and the tuning characteristic of the $TE_{01\delta}$ mode can be determined with an accuracy which is satisfactory for most engineering purposes.

The realizations of dielectric resonator TEM line cavities have demonstrated the possibility of combining the electrical performance of waveguide cavities with small size and simplicity of TEM line circuits. It seems that miniature tunable X-band resonators having Q factors of about 3000 and a frequency stability better than $3\times 10^{-6}/°C$ could be produced for the price of a capacitor.

Acknowledgment

The author wishes to thank Prof. A. K. Smoliński of Warsaw Technical University, Warsaw, Poland, for his support and encouragement during the course of this work, Prof. M. Schetzen of Northeastern University, Boston, MA, for helpful comments, and also anonymous reviewers, whose remarks greatly improved the presentation of the results obtained.

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